Optimal Location of Vertical Wells: Decomposition Approach

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The generation of a reservoir development plan with well locations, given a reservoir property map and a set of infrastructure constraints, represents a very challenging problem. The problem of selecting the optimal vertical well locations was formulated with 3-D information as a MILP problem where the binary variables correspond to the decisions of well locations and vertical well completions. A novel decomposition-based solution procedure is proposed to address real fields characterized by 60,000 grid points. The approach is based on the ideas of refining the feasible set of candidate well locations by applying quality cut constraints. For the solution of the subproblems required at each iteration, a model reformulation was then suggested that enables the efficient determination of the optimal well locations. Two industrial case studies are considered to illustrate the applicability of the proposed procedure to realistic large-scale problems.

Introduction

The problem of developing an offshore field involves the selection of: number and locations of platforms; number and locations of wells; production system; and scheduling of facility and well production.

Although these decisions are highly dependent on each other, their simultaneous consideration in an optimization problem would result in an intractable problem. A lot of research work appeared in the literature that deals with drilling technology, but much less attention has been paid in the area of optimization of facility and well location, as well as production optimization.

Representative research efforts for the location-allocation problem following a mathematical programming approach include those of Devine and Lesso (1972), Dogru (1987), Grimmet and Startzman (1988), and Sullivan (1982). Almost all of the proposed models give rise to large problems that cannot be solved to optimality. Heuristic solution procedures were proposed that do not guarantee global optimality. Recently, Garcia-Diaz et al. (1996) proposed a new methodology based on a branch and bound procedure using Lagrangian relaxation techniques to generate valid tight lower bounds. They presented an example for an offshore field development for the solution of which they considered different scenarios which greatly reduced the size of the problem since each scenario was considered independently and resulted in MILP problems up to the size of 2,760 binary variables and 6,329 constraints.

For the production optimization given a set of existing wells, Lasdon et al. (1986) proposed a nonlinear optimization model to determine the optimal flows. The problem of determining the optimal well locations was addressed by Rosenwald and Green (1973) who employed a MILP approach to select the well sites that accomplish production targets from a predetermined subset of locations. They assumed a specified production vs. time relationship for the reservoir considered and a set of possible locations for the new wells. The algorithm then selected a specified number of wells from the candidate locations and determined the proper sequence of rates from the wells. They demonstrated their approach through example problems considering a small number of possible well locations.

Seifert et al. (1996) presented a well placement method based on a geostatistical reservoir model. They performed an

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exhaustive search for a large number of candidate well trajectories from a platform location, with a preset radius, inclination angle, well length, and azimuth. Then, the reservoir quality along each well trajectory was analyzed statistically with respect to intersected net pay or lithology to find the highest expected value of net pay for a set of wells. The placement location of candidate wells was not a variable in the analysis; thus, the procedure finds a statistically local maximum rather than a global optimum.

Vasantharajan and Cullick (1997) proposed a MILP formulation for determining the optimal location of wells considering 2-D information about the productivity of the reservoir and connectivity points. They addressed large-scale problems in reasonable computational time, and they incorporated platform location constraints assuming that the location of the platform is known.

Iyer et al. (1998) proposed a multiperiod MILP model for the planning and scheduling of investment and operation in offshore oil field facilities. The objective was to optimize the net present value for a given planning horizon taking into account the choice of reservoirs to develop, the well drilling schedule, the capacities of wells and production platforms, and the fluid production rates from wells for each time period. A sequential decomposition strategy was followed for the solution of the resulting formulation.

In this article, 3-D information about the field is employed that includes point quality and geo-object, which expresses the productivity and connectivity of the point. Based on this information, the problem of selecting the optimal location of vertical wells is formulated as a MILP optimization problem as discussed in detail in the section on mathematical formulation, where three models are suggested based on the field specificity. A decomposition based approach is proposed in order to address realistic problems based on the application of quality cut-off constraints. Two industrial case studies are presented to illustrate the applicability and effectiveness of the proposed approach.

Problem Definition

This article addresses the well site selection issues faced by a reservoir management team during a project development for which the wells are sited to maximize productivity while satisfying constraints. The reservoir productivity at this stage is a static metric of the reservoir productivity, such as a net pay, permeability-thickness, or a combination. The focus is on modeling the spatial location, taking into account interwell spacing, distance from platforms, and well configurations. Subsequent detailed flow simulation is warranted to validate the selection and to determine the appropriate production policy from these wells to meet desired production targets.

In particular, the problem considered can be described in the following way. Given is a specific field which is discretized considering a set of grid points. Each point in the grid corresponds to a potential well location. In addition, grid points are considered in the z direction representing the potential completions if a drill is decided at this point (see Figure 1).

Each (x, y, z) point is characterized by its *quality* representing the productivity of this point (see Vasantharajan and Cullick, 1997) and the *geo-object* at which this point belongs representing the connectivity of this point with the other potential well points. The fact that different grid points belong to the same geo-object poses the restriction of well distance. This means that potential wells have to be separated from each other by $D^s_{\min}$ or $D^d_{\min}$ if the points belong to the same or different geo-objects, respectively, where $D^s_{\min} > D^d_{\min}$.

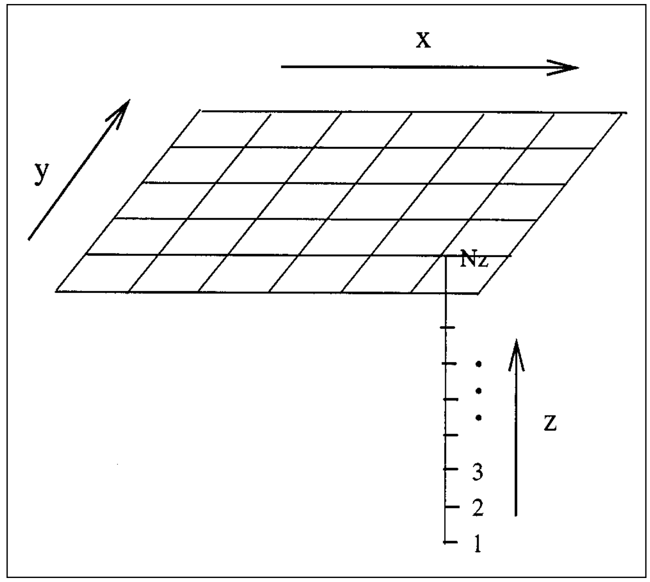

Figure 1. *z* direction grid points.

Given the above information for the considered field, the objective is to determine the optimal location and/or number of wells so as to maximize the overall quality of production. In a second stage, the goal is the incorporation of cost data so as to identify the trade-offs between drilling cost and productivity, and determine the economically optimal well location that maximizes production quality.

In the next section, three different mathematical models are proposed to address the problem of optimizing the potential well locations so as to maximize quality as a measure for the productivity. Cost is also studied in an attempt to identify the trade-offs between quality and cost although the appropriate cost parameters expressing the cost to the production relationship are unknown.

Mathematical Formulation

Model 1

In this model the different possible completions in the z direction are represented by different variables. This means that this model is suitable for problems where the quality changes with respect to z even if the points belong to the same geo-object. The mathematical model involves the following indices, sets, parameters, and variables:

Indices. i, j potential well location points; k potential points for well completions in the z direction.

Sets. N all potential well locations in the x-y plane; K all potential well completion in z direction; I_{ex} existing well locations.

Parameters. x_i, y_i x, y coordinates of point i in grid units; z_i^k z coordinate of point (i, k); $D_{\min}^s$ denotes the minimum distance required between wells in the same geo-object; $D_{\min}^d$ denotes the minimum distance required between wells in different geo-objects; N_{com} denotes the maximum number of completions allowed per well; q_i^k quality of point z_i^k; α cost parameter.

Variables. $yv(x_i, y_i)$ binary variables expressing the existence of a well at the point (x_i, y_i); $zv^k(x_i, y_i)$ binary variables expressing the completion at point z_i^k if a well is decided at point (x_i, y_i); $d(i, j)$ distance between point (x_i, y_i) and (x_j, y_j); W_{on} total number of wells; Q total quality of wells; C total cost of wells.

Based on this notation the mathematical model for the optimal vertical well location problem involves the following constraints:

Minimum Distance Constraints: Points Belonging to the Same Geo-Object.

$$yv(x_i, y_i) + yv(x_j, y_j) \le 1, \quad \forall i, j \in N, \quad d(i, j) < D_{\min}^s \quad (1)$$

These constraints express the requirement that potential well locations should be separated by $D_{\min}^s$ if they belong to the same geo-object. The set N involves all potential well locations in the field. $d(i, j)$ is the distance between location i and location j represented by either the Euclidean or rectilinear metric, as will be discussed later in this section.

Note that an alternative way to impose constraint 1 is the following

$$\sum_{j \in N,\, d(i,j) < D_{\min}^s} (1)\, yv(x_i, y_i) + \sum_{j \in N,\, d(i,j) < D_{\min}^s} \left[yv(x_j, y_j) \right] \le \sum_{j \in N,\, d(i,j) < D_{\min}^s} (1), \quad \forall i \in N.$$

This corresponds to the summation of constraints 1 over (j) such that for given (i) $d(i, j) < D_{\min}^s$ and the potential well locations belong to the same geo-object. Case study 2 discussed in a later section illustrates the applicability of these constraints and the effect on the computational performance of the solution procedure.

Minimum Distance Constraints: Points Involving Different Geo-Objects.

$$yv(x_i, y_i) + yv(x_j, y_j) \le 1, \quad \forall i, j \in N, \quad d(i, j) < D_{\min}^d \quad (2)$$

These constraints express the requirement that potential well locations should be separated by $D_{\min}^d$ if they involve different geo-objects, where $D_{\min}^s > D_{\min}^d$.

Minimum Distance Constraints: Existing Wells.

$$yv(x_i, y_i) + yv(x_j, y_j) \le 1, \quad \forall i \in N, \quad \forall j \in I_{\text{ex}}, \quad d(i, j) < D_{\min}^s$$

$$yv(x_i, y_i) + yv(x_j, y_j) \le 1, \quad \forall i \in N, \quad \forall j \in I_{\text{ex}}, \quad d(i, j) < D_{\min}^d \quad (3)$$

where I_{ex} is the set of existing well locations. These constraints express the requirement that potential well locations should be $D_{\min}^s$, $D_{\min}^d$ from the already existing wells if they belong to the same and different geo-objects, respectively.

Maximum Completion Constraints.

$$\sum_k zv^k(x_i, y_i) \le N_{\text{com}}\, yv(x_i, y_i), \quad \forall i, j \in N \quad (4)$$

$$zv^k(x_i, y_i) \le yv(x_i, y_i), \quad \forall i, j \in N \quad (5)$$

These constraints express the requirement that the maximum number of completions per well is N_{com}. Moreover, if the well does not exist at the point (x_i, y_i), then all the $zv^k(x_i, y_i)$ binary variables that correspond to this point should also take the value of zero. Constraints 5 are included since they improve the computational performance.

Total Number of Wells Constraints.

$$\sum_i yv(x_i, y_i) = W_{\text{on}} \quad \forall i \in N \quad (6)$$

This constraint restricts the total number of wells.

Total Quality of Wells Constraints.

$$\sum_{i \in N} \sum_{k \in N_{\text{com}}} zv^k(x_i, y_i) q_i^k = Q \quad (7)$$

This constraint evaluates the quality of the optimal well locations.

Total Cost of Wells.

$$\sum_i \frac{\alpha}{2} yv(x_i, y_i) + \frac{\alpha}{2} \sum_i \sum_k zv^k(x_i, y_i) = C \quad \forall i \in N \quad (8)$$

This constraint evaluates the cost of the optimal well locations accounting for the cost of completions. The parameter α accounts for the trade-off between cost and productivity. It is determined based on the observed quality data and the cost of drilling from previous industrial case studies.

Note that the above model can provide the basis for placing deviated wells since the z dimension is treated explicitly in the formulation. Deviated wells are most commonly used to increase the field productivity.

Model 2

In this model, the points in the z direction belonging to the same geo-object are considered together and are represented by one variable and an average quality. This means that this model is more suitable for problems where the quality within the same geo-object does not change very much. This formulation has the advantage of involving fewer variables since the variables in the z direction are equal to the ones corresponding to different geo-objects, which are much less than the different z points. However, this formulation cannot address the problem with deviated wells since only average z values for the different geo-objects are considered.

In both models 1 and 2 the distance requirement between potential wells is expressed in terms of the Euclidean distance, that is, the potential well at position (x_i, y_i) should be at least $D^s_{\min}$ from a different well (x_j, y_j) if they both belong to the same geo-object, which means

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geq D^s_{\min}$$

and at least $D^d_{\min}$ if they are different geo-objects, where $D^s_{\min} > D^d_{\min}$.

The mathematical formulation in this case has the same form except that the variables $z^k(x_i, y_i)$ correspond to different geo-objects in the z direction and not the different z points.

Model 3

In this model the distance constraints are written in terms of zv variables rather than the yv variables as it is the case for Models 1 and 2.

Minimum Distance Constraints: Points Belonging to the Same Geo-Object.

$$zv^k(x_i, y_i) + zv^k(x_j, y_j) \leq 1, \quad \forall i, j \in N, \quad \forall k \in K$$
$$d(i, j) < D^s_{\min} \quad (9)$$

These constraints express the requirement that potential well completions should be separated by $D^s_{\min}$ if they belong to the same geo-object. The set N involves all potential well locations in the field. $d(i, j)$ is the distance between location i and location j represented by either the Euclidean or rectilinear metric, as will be discussed later in this section.

An additional set of constraints could also be introduced that have the following form

$$zv^k(x_i, y_i) + \sum_{j|d(i,j) < D^s_{\min}/2} zv^k(x_j, y_j) \leq 1, \quad \forall i \in N, \quad \forall k \in K \quad (10)$$

These constraints express the requirement that only one of the points in the inner region of Figure 2 can be selected. As will be shown in Case Study 2, the incorporation of this set of constraints enhances the computational performance of the solution approach.

Note, that expressing the distance constraints in terms of the z variables increases the size of the mathematical formulation but on the other hand allows for more flexibility on selecting the optimal well locations and completions and better trade-off between productivity and cost as it will be shown through the second case study where this model is applied.

Alternative distance metrics

In all the presented models the distance constraints between location i and location j could be expressed through the Euclidean distance constraints, as, for example, for Eq. 1

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \leq D^s_{\min}$$

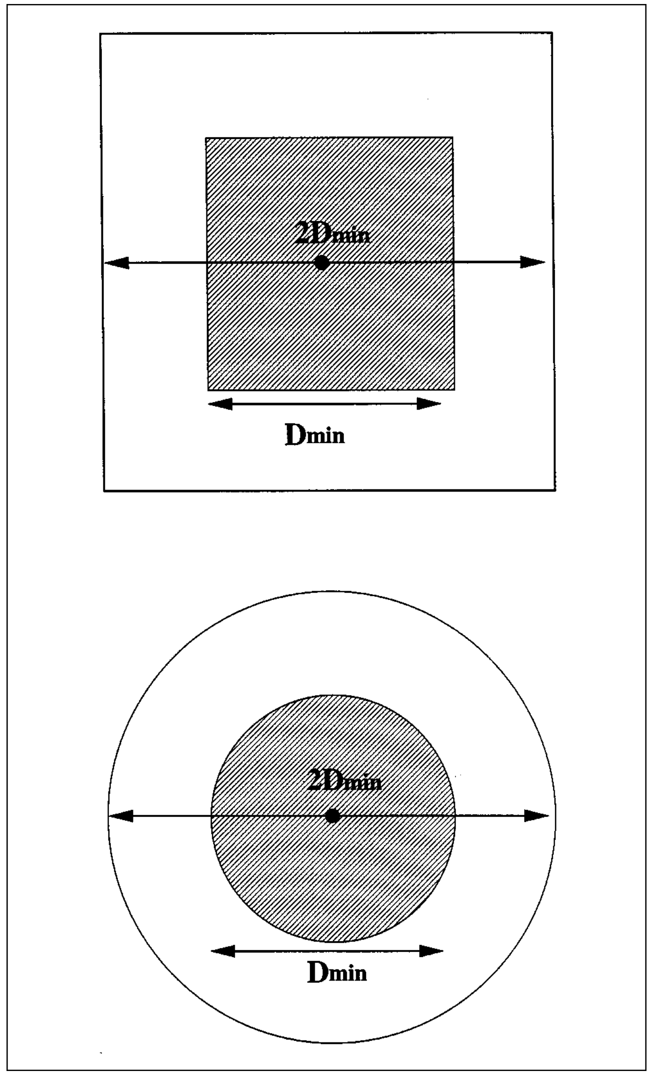

Figure 2. "Clique" distance requirements.

or the rectilinear distance constraints

$$|x_i - x_j| \leq D^s_{\min}$$

$$|y_i - y_j| \leq D^s_{\min}$$

The Euclidean distance assumption is equivalent to the circular shape of the exclusion region around the considered grid point. The rectilinear distance representation corresponds to the assumption that the "no-flow" boundaries between well sites of similar producing characteristics will occur at the mid-point of the distance between them, which means geometrically a square-shaped exclusion zone of side grid units around a grid cell (see Figure 3).

Note that the rectilinear distance requirements are more restrictive than the Euclidean distance requirements excluding more potential well locations, namely, the points outside the circle with radius $D^s_{\min}$ and inside the square with half side equals $D^s_{\min}$ (see Figure 3). The consequences of this

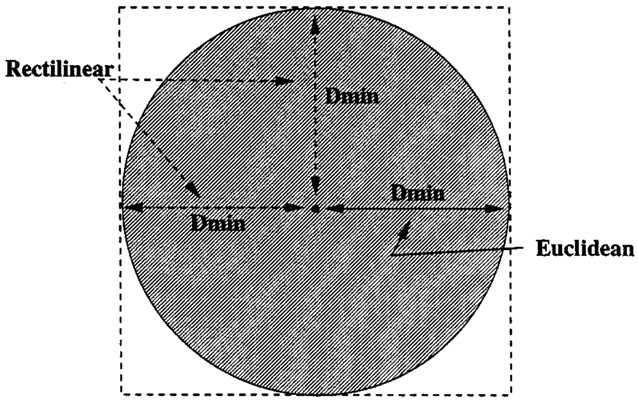

Figure 3. Different distance requirements.

remark will be illustrated through the case studies considered in the next sections where the rectilinear constraints lead to more conservative results in terms of the productivity and overall objective.

All the proposed models are mixed-integer linear programming problems with the binary variables representing the location of wells $y(x_i, y_i)$ and the vertical well completions $z^k(x_i, y_i)$.

Proposed Approach

Decomposition procedure

A systematic decomposition approach is presented for the prediction of the optimal location and/or the optimal number of wells in order to maximize the overall quality of the production. The proposed approach follows a decomposition procedure based on the number of potential well locations examined and involves the following steps which are also shown in Figure 4:

Step 1: Rank all the candidate points in terms of the maximum over z direction quality. Apply a quality cut-off constraint and consider the remaining points.

Step 2: The overall region under consideration is first decomposed in different regions based on the distribution of the potential well locations.

Step 3: Based on the model considered (that is, Model 1, 2, or 3) the resulting MILP optimization problem is solved for each one of these regions (see the next subsection).

Step 4: The optimal well locations determined are ranked in terms of their quality.

Step 5: A quality cut-off constraint is then applied and the locations above the specified quality cut-off were considered fixed. A feasibility problem is solved (see the feasibility test subsection) for the whole region and the remaining feasible points are identified that satisfy the requirement of being at least $D^s_{\min}$ away from the already fixed locations if they belong at the same geo-object or $D^d_{\min}$ if they belong at different geo-objects.

Step 6: Using a quality cut-off for the feasible points, the remaining points are then considered in the optimization problem and the optimal well locations are determined for this set of points.

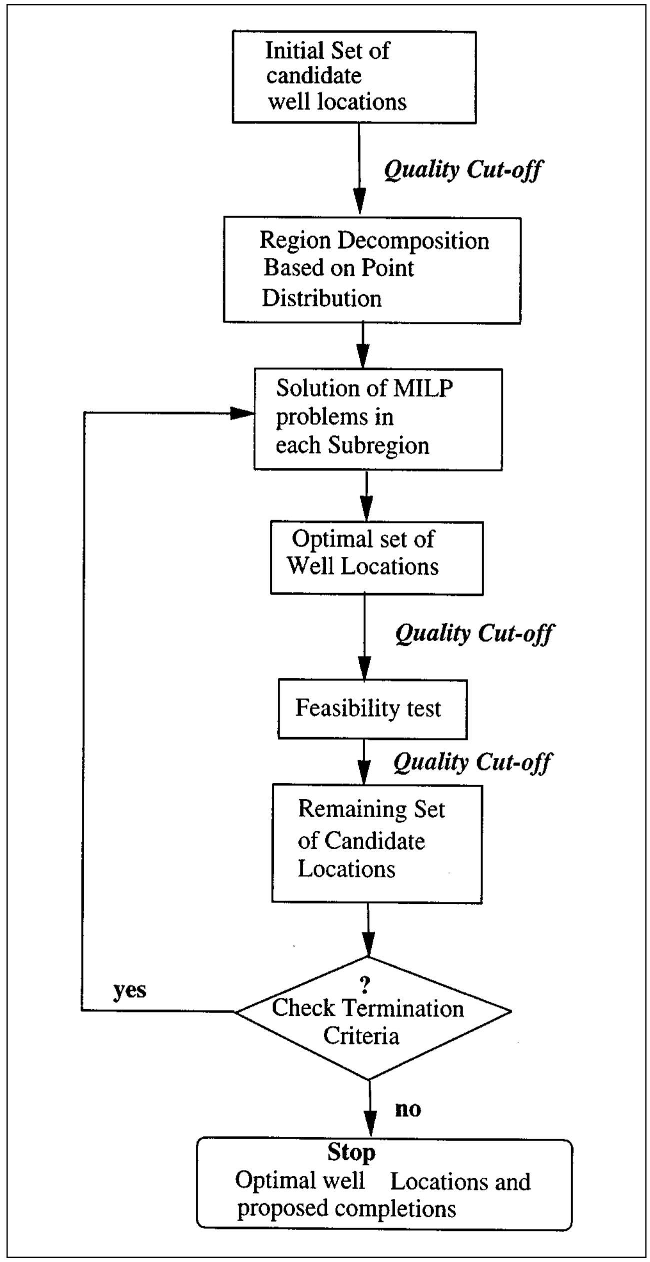

Figure 4. Proposed approach.

Step 7: The termination criteria for the algorithm are the following:

(a) The overall number of the obtained wells exceeds the required wells.

(b) There are no additional wells that can be placed. This can be verified by fixing the determined wells and solving the feasibility problem. If no points remain it means that there are no potential well locations left.

If one of the above criteria is met then the algorithm stops; otherwise it continues iterating from Step 5.

Remark 1: Note that at Step 5, when solving the feasibility test problem all the points are considered and not only those above the quality considered at Step 1. Consequently, as iterations continue the initial set of points is enlarged.

Remark 2: An alternative to the above presented decomposition approach will be to apply the cut-off procedure in each subregion separately and then combine the obtained optimal well locations by solving the feasibility subproblem to eliminate any distance violations due to decomposition of the problem in Step 1. The application of this alternative solution procedure is illustrated in the second case study in the next section.

Solution of subproblems

As mentioned earlier, the mathematical models presented in the previous section correspond to MILP problems that can be solved employing standard solution techniques based on branch and bound algorithmic procedure implemented in commercially available software packages like CPLEX and OSL. Note, however, that in all the proposed models the resulting formulation corresponds to a large-scale optimization problem involving 430,505 constraints and 2,000 binary variables for the case where 2,000 well locations and an average over z quality (models 2, 3) are considered. The consideration of 10,000 surface points corresponds to only 25% of the potential well locations on the field considered in the first case study of the next section, gives rise to an intractable size model with 10,000 binary variables for models 2 and 3. The incorporation of the z direction data points, as in Model 1, results in even larger models. For the case of 2,000 potential surface well locations and 65 candidate completion points, Model 1 involves 438,889 constraints and 8,383 binary variables. Note that not all the 65 different z locations correspond to potential completion candidates but only the ones that belong to different geo-objects.

Employing a Branch and Bound framework for the solution of MILP optimization problem, the nodes that need to be examined in the worse case scenario are 2^N where N are the binary variables, which suggest that for a problem involving 2,000 surface points considering 65 vertical points we need to consider 2^{8383} nodes to solve it to optimality.

Feasibility test

At this step, the initial set of candidate well locations are examined regarding their feasibility with respect to the existing wells, as well as the locations of wells selected so far in the iterative procedure described in the subsection on decomposition procedure. This step is applied automatically by checking the viability of the distance constraints for each one of the candidate well locations. Depending on the distance metric used, the distance constraints have the following form:

Euclidean

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geq D^s_{\min},$$

$$\forall i \in N, \quad j \in I'_{ex}, \quad \text{geo}(i,k) = \text{geo}(j,k) \quad \forall k \in K_i \quad (11)$$

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geq D^d_{\min},$$

$$\forall i \in N, \quad j \in I'_{ex}, \quad \text{geo}(i,k) \neq \text{geo}(j,k) \quad \forall k \in K_i \quad (12)$$

Rectilinear

$$|x_i - x_j| \geq D^s_{\min} \vee |y_i - y_j| \geq D^s_{\min},$$

$$\forall i \in N, \quad j \in I'_{ex}, \quad \text{geo}(i,k) = \text{geo}(j,k) \quad \forall k \in K_i \quad (13)$$

$$|x_i - x_j| \geq D^d_{\min} \vee |y_i - y_j| \geq D^d_{\min},$$

$$\forall i \in N, \quad j \in I'_{ex}, \quad \text{geo}(i,k) \neq \text{geo}(j,k) \quad \forall k \in K_i \quad (14)$$

where I'_{ex} is the augmented set of existing well locations involving the locations of the wells selected.

The application of the above constraints requires checking the connectivity of the points considered which is represented by the corresponding geo-object. Consequently, the feasibility test consists of the following two-step procedure for each candidate location.

Step A: Check the geo-objects of the candidate points with the set of existing and selected well locations.

Step B: For the points that belong to the same geo-object, apply constraints (Eq. 11 or Eq. 13) based on the model used. For the points that do not belong in the same geo-object, apply constraints (Eq. 12 or Eq. 14), based on the model used.

Step C: If a point does not satisfy the distance constraints in Step B, then it is not considered further as potential well location.

Application of the Proposed Approach

Case study 1

The proposed approach is applied to a specific field considering Models 1 and 2. For Model 2, both Euclidean and rectilinear distance constraints are used. The project area for this field was established as a hyper-rectangular area with dimensions of 48,070 ft × 14,960 ft × 1,105 ft. Within this space, a Cartesian grid was defined with cells 110 ft × 110 ft × 17 ft (see Figure 5). This grid consists of 59,432 candidate surface well locations for which 65 possible well completions are allowed based on the grid point connectivity represented by point geo-object number (see Figure 6).

In this section the details of the application of the proposed approach are presented for Model 1, while only the

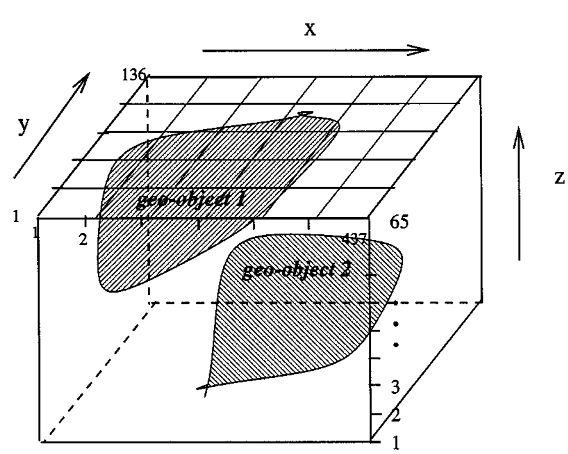

Figure 5. Potential well locations.

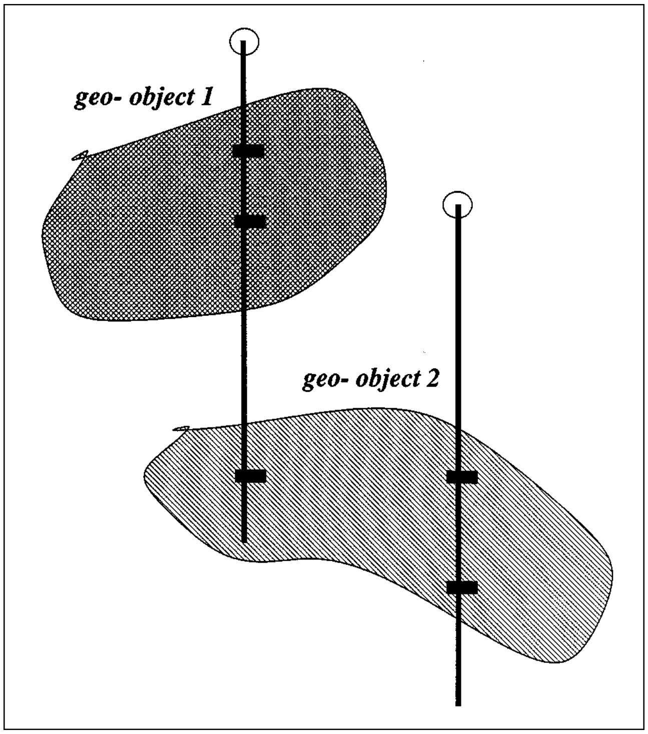

Figure 6. Different well completions.

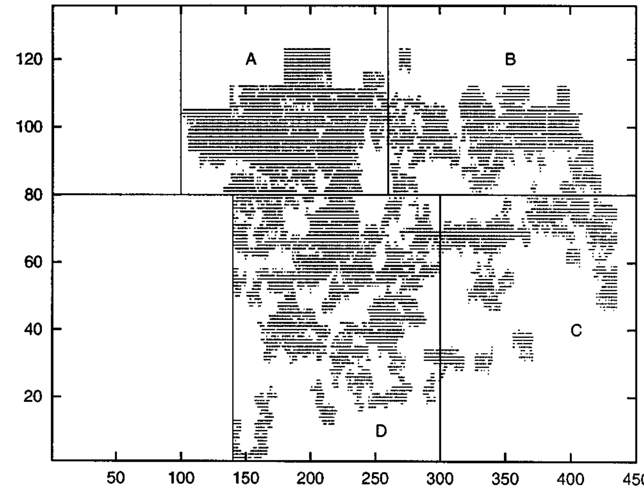

Figure 8. Field decomposition.

Table 1. Boundaries of the Different Regions

Region	x Bounds (Grid Units)	y Bounds (Grid Units)	No. of Potential Well Locations
A	$100 \leq x \leq 260$	$80 \leq y \leq 137$	4,129
B	$260 \leq x \leq 436$	$80 \leq y \leq 137$	2,795
C	$300 \leq x \leq 436$	$0 \leq y < 80$	1,991
D	$140 \leq x \leq 300$	$0 \leq y < 80$	5,328

results for Model 2 are provided using both Euclidean and rectilinear distance constraints. The plot of all potential well locations for this field is shown in Figure 7. Based on the distribution of the points, the field is decomposed into four different regions as shown in Figure 8. Note that not all the points are considered at this decomposition stage, but the regions with the largest number of points and high quality are considered. The boundaries of each region and the number of potential well locations are shown in Table 1.

Using Model 1 in the Step 3 where all the z points are considered explicitly as binary variables representing potential well completions, the resulting optimal well locations that maximize the production quality are shown in Figure 9. The results are summarized in Table 2. Note that, in order to restrict the size of the resulting model, a maximum of 2,000 potential surface points are considered at each region. However, the MILP models still correspond to very large optimization problems since, for example, for region A, the MILP model involves 8,383 binary variables and 438,889 constraints.

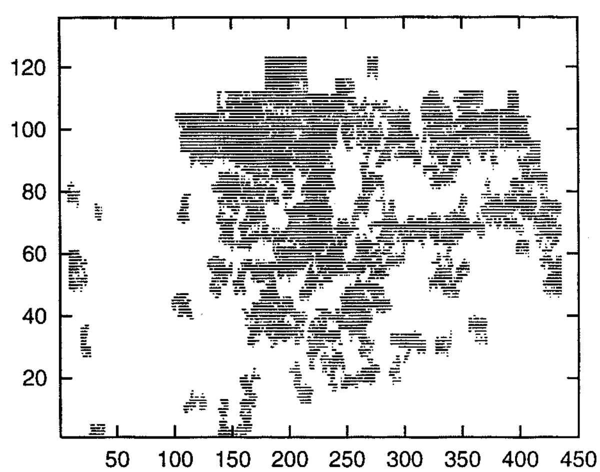

Figure 7. Potential well locations.

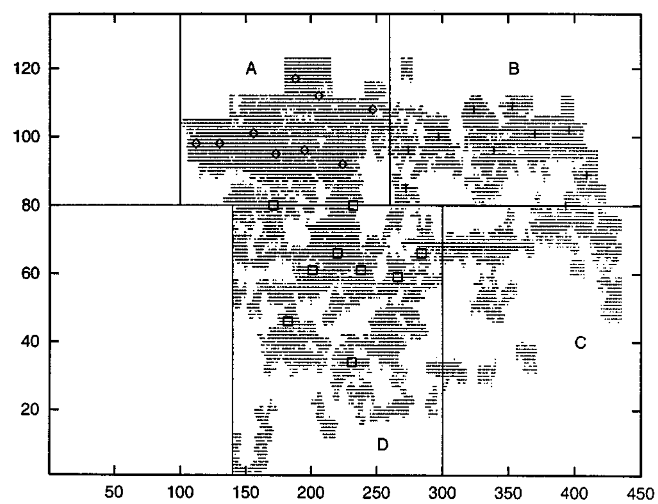

Figure 9. Optimal well locations.

Table 2. Optimal Solution for the Different Regions

Region	No. of Wells	Overall Quality	Average Quality
A	9	98,521.052	21,282.5832
B	10	73,548.36	16,463.36
C	9	54,945.883	11,797.683
D	9	81,060.393	17,905.657

Table 3. Optimal Well Locations

Well	X Coord.	Y Coord.	No. of Completions	Overall Quality	Average Quality	Geo-Object
1	147.0	85.0	5	7,631.379	1,526.276	1.0
2	394.0	76.0	5	7,428.15	1,485.630	3.0
3	427.0	56.0	5	7,100.81	1,485.130	8.0
4	197.0	34.0	5	7,405.685	1,481.137	1.0
5	266.0	78.0	5	7,007.868	1,401.574	5.0

Note that, in the above analysis, a maximum of five completions is allowed per well. Since, in this phase, the objective is the maximization of quality, this results in selecting all the five completions if this is possible.

The optimal locations of wells for each region are then sorted, and an additional quality cut-off of 1,500 is applied. This means that only the locations with maximum over z quality above 1,500 are considered fixed, which results in 28 well locations, 9 from area A, 5 from area B, 2 from region C, and 9 from region D plus 3 that already exist.

Then, the feasibility step is applied fixing the 28 well locations obtained so far. This means that the points with $d(i,j) \leq D^s_{\min}$ around those points, if they belong in the same geo-object, and the points with $d(i,(j) \leq D^d_{\min}$, if they belong to different geo-objects. The remaining potential well locations are then sorted, and a cut-off quality of 1,000 based on maximum over z quality is applied. The remaining 1,047 points are shown in Figure 10. Considering these points as the potential well locations in the optimization model, the additional optimal well locations are determined. The optimal well locations that feature a maximum quality above 1,400 are the locations shown in Table 3. There are five new wells identified and, hence, we have a total of 33 wells. The feasibility step is then applied fixing all the 33 optimal well locations. The remaining feasible points are obtained, sorted, and a quality cut-off of 700 in terms of maximum over z quality is applied. The remaining 1,658 potential well locations are shown in Figure 11.

These points are then considered in the optimization problem that results after a quality cut-off of 1,000 is applied in

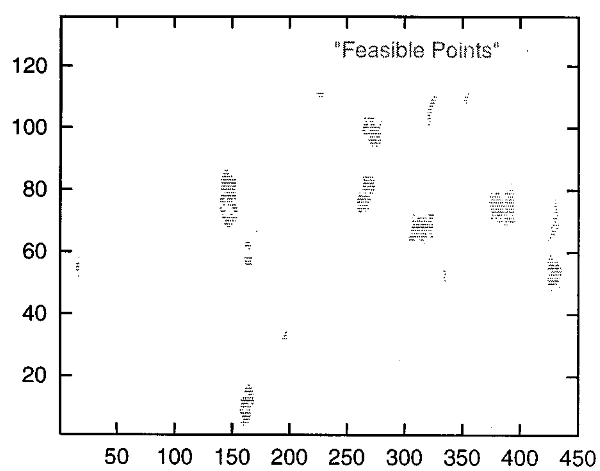

Figure 10. Feasible potential well locations.

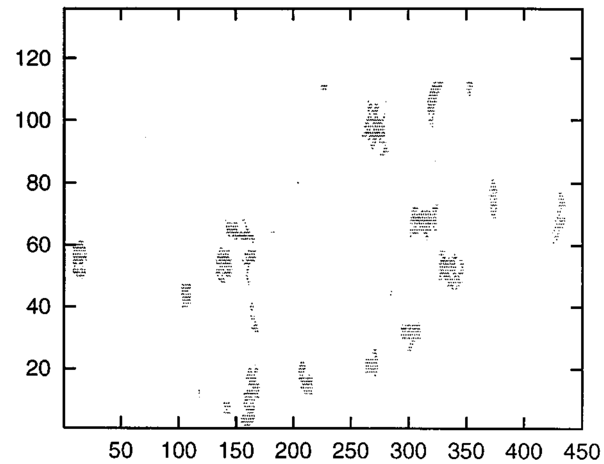

Figure 11. Feasible potential well locations.

the optimal wells, in the following 10 optimal well locations as shown in Table 4.

In the final step all the locations determined so far are further checked for feasibility, especially the wells at the areas near the region boundaries. For the selected points, it is found that in three cases the distance requirement is violated as illustrated in Figure 12. The points represented by squares have a distance 14.422 and belong to geo-object 1, the points represented by cross have a distance of 15.133 and belong to geo-object 1, and the points indicated with an "x" have a distance of 11.402 and belong to geo-object 3.

For these three cases, only the location with the maximum overall quality is considered fixed, whereas the other two are rejected. After this additional step, we end up with 40 selected well locations. The feasibility test is then performed

Table 4. Optimal Well Locations

Well	X Coord.	Y Coord.	No. of Completions	Overall Quality	Average Quality	Geo-Object
1	160.0	11.0	5	6,797.368	1,359.474	7.0
2	227.0	110.0	4	5,306.672	1,326.668	1.0
3	426.0	66.0	5	6,260.815	1,252.163	3.0
4	353.0	109.0	3	3,560.895	1,186.965	4.0
5	324.0	108.0	4	4,634.588	1,158.647	4.0
6	165.0	63.0	5	5,757.215	1,151.443	1.0
7	274.0	96.0	5	5,743.301	1,148.660	1.0
8	376.0	77.0	4	4,436.288	1,109.072	3.0
9	319.0	67.0	5	5,503.27	1,100.654	1.0
10	335.0	53.0	5	5,077.42	1,015.484	6.0

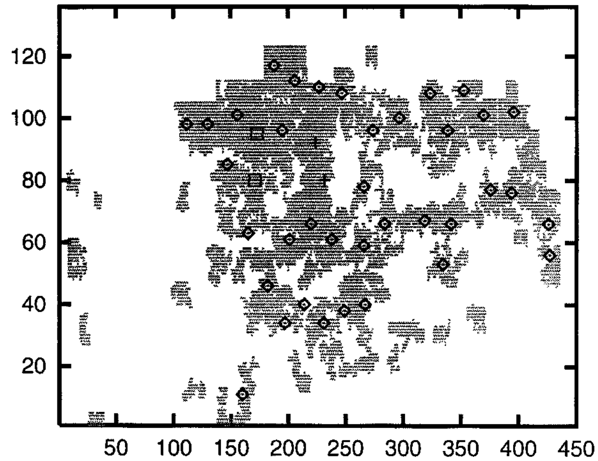

Figure 12. Infeasible points.

and the feasible points are determined and considered in the optimization framework in order to determine 30 more required well locations. The optimal 30 well locations are shown in Table 5.

In conclusion the optimal well locations obtained are shown in Figure 13.

Following the same steps, but considering Model 2 using Euclidean and rectilinear distance constraints, respectively, the optimal well locations shown in Figures 14 and 15 are obtained. Due to strict feasibility constraints, the utilization of the rectilinear distance constraints results in the selection of 62 instead of 70 well locations.

The production qualities of the optimal wells determined from the solution of Models 1 and 2 are shown in Table 6. Note that, in terms of the overall average quality, Model 2 outperforms in comparison with Model 1. This happens due to the different objective of Model 1 that targets on maximizing the summation of all the optimal wells considering all well completions. This means that Model 1 would select a point with larger overall quality compared to a point with smaller quality but higher average quality. Moreover, note that Model 2 with the use of the rectilinear distance constraints results in smaller average quality due to tighter feasibility requirements (see the Problem Definition section).

Finally, note that all optimization models result in higher average quality than the proposed ones as shown in Tables 6 and 7. Figures 16, 17 and 18 illustrate the proposed well location in comparison with the optimal wells suggested by Models 1, 2, and 3, respectively. The proposed wells shown in Figures 16, 17 and 18 correspond to the locations selected by the industrial team based on the observation of geophysical data, which was the realistic well selection procedure prior to this study.

Note that, although in Step 3 of the proposed decomposition approach the sorting of the proposed optimal well locations is based on the maximum quality of the point in the z direction, an average quality, the overall summation of qualities can also be considered.

The feasibility stage of Step 4 can also be applied to identify the feasible points in the case where different well locations are fixed, for example, in the case that there are already existing wells avoiding in this way the incorporation of additional constraints in the optimization problem [constraints (Eq. 3) in the formulation].

Table 5. Optimal Well Locations

Well	X Coord.	Y Coord.	No. of Completions	Overall Quality	Average Quality	Geo-Object
1	296.0	98.0	5	2,755.555	551.111	17.0
2	211.0	16.0	5	4,886.605	977.321	13.0
3	167.0	35.0	5	4,800.65	960.130	1.0
4	180.0	77.0	2	1,908.776	954.388	1.0
5	139.0	53.0	5	4,734.31	946.862	11.0
6	13.0	57.0	5	4,593.075	918.615	12.0
7	202.0	79.0	2	1,805.772	902.886	1.0
8	144.0	67.0	5	4,405.47	881.094	1.0
9	266.0	19.0	5	4,317.73	863.546	9.0
10	106.0	41.0	5	4,100.17	820.034	14.0
11	285.0	45.0	5	3,968.205	793.641	1.0
12	118.0	13.0	5	3,435.994	687.199	15.0
13	294.0	32.0	5	3,505.97	701.194	10.0
14	362.0	38.0	5	3,234.55	646.910	18.0
15	312.0	32.0	4	2,461.812	615.453	10.0
16	362.0	69.0	5	3,048.02	609.604	1.0
17	34.0	4.0	5	2,970.265	594.053	20.0
18	142.0	2.0	5	2,797.82	559.564	16.0
19	21.0	33.0	5	2,648.915	529.783	22.0
20	353.0	56.0	4	2,027.54	506.885	6.0
21	248.0	19.0	5	2,487.345	497.469	9.0
22	331.0	29.0	4	1,903.424	475.856	19.0
23	13.0	80.0	5	2,376.785	475.357	23.0
24	400.0	63.0	5	2,350.39	470.078	21.0
25	106.0	72.0	5	2,317.22	463.444	25.0
26	318.0	88.0	5	2,302.312	460.462	4.0
27	293.0	23.0	5	2,280.125	456.025	24.0
28	238.0	16.0	4	1,662.392	415.598	1.0
29	277.0	122.0	5	2,034.03	406.806	27.0
30	33.0	74.0	5	1,933.81	386.762	28.0

Case study 2

For this case, the decomposition method is applied to a field where the simultaneous consideration for all points is also possible within the optimization framework so as to compare and evaluate the results of the proposed approach.

Decompositon Approach: Original Distance Constraints. The grid considered for this case study consists of 6,206 candidate surface well locations for which 75 possible well completions are allowed based on the grid point connectivity represented by point geo-object number (Figure 19).

Based on the distribution of the points, the field is decomposed into three different regions as shown in Figure 20. The boundaries of each region and the number of potential well locations are shown in Table 8.

The alternative solution procedure, discussed in the subsection on decomposition procedure, utilizing rectilinear distance constraints is applied here where the cut-off procedure is applied to each subregion separately. Model 3 is considered since the incorporation of the clique constraints leads to the reduction of the integrality gap of the MILP models and, thus, enables the determination of better solutions. A mini-

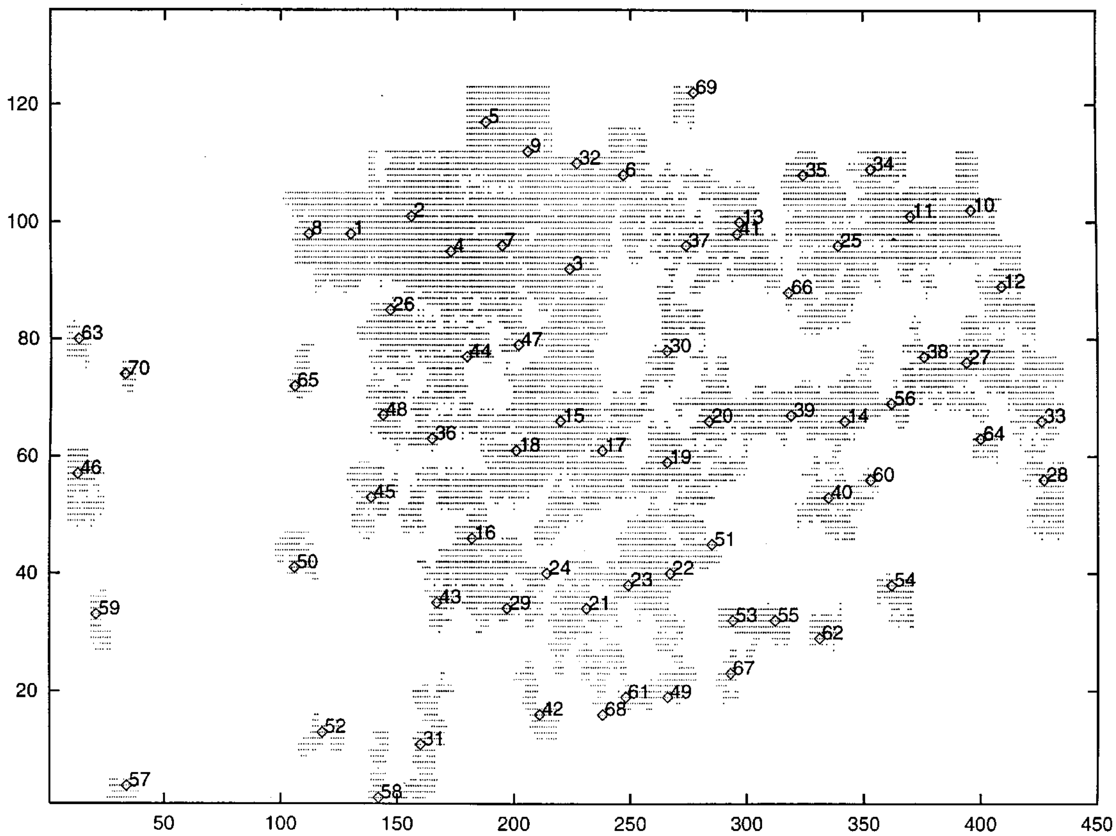

Figure 13. Model 1: optimal well locations.

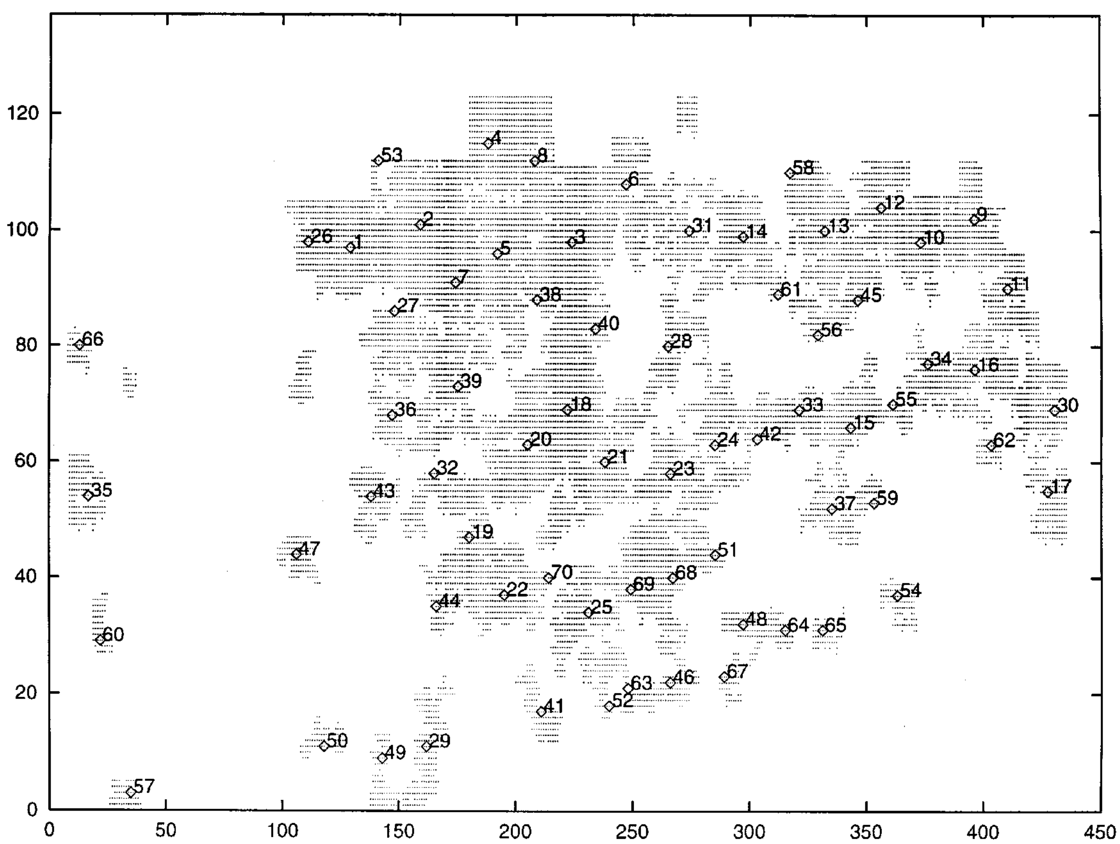

Figure 14. Model 2: optimal well locations—Euclidean distance.

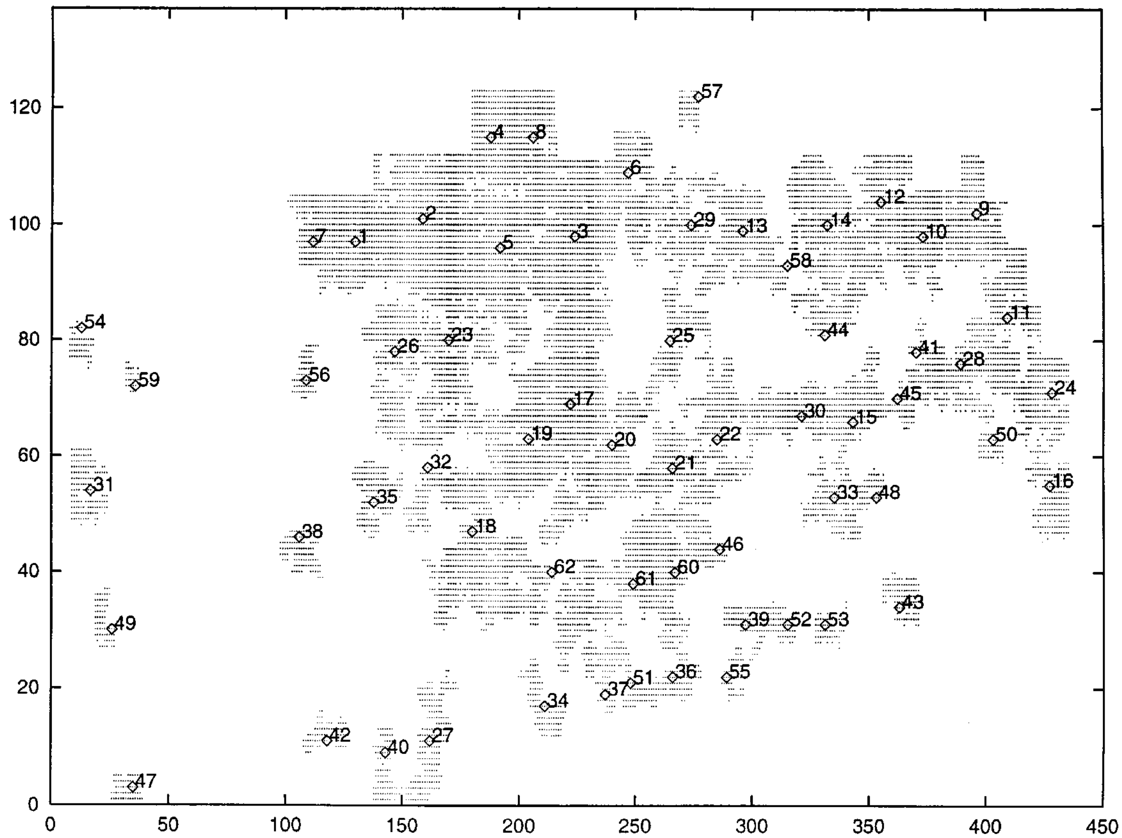

Figure 15. Model 2: optimal well locations—rectilinear distance.

mum distance requirement of 18 grid points has to be met. The following steps are involved in the solution procedure:

- Since all the subregions A, B, and C involve a fairly small number of potential locations, there is no need of applying a quality cut-off for these regions. The determination of optimal well locations for these regions lead to 12 optimal locations in area A, 19 in area B, and 10 in area C.
- The results for each region are shown in Table 9.
- Consider the $12+19+10=41$ optimal wells found within the feasibility test formulation to identify the feasible optimal well locations; 39 optimal wells and completions are identified.
- Fix the 39 optimal wells and determine the remaining feasible potential sites. Three feasible points have been found with quality smaller than 0.8 units, which is the cost of drilling a well. Consequently, no more well locations can be selected.

The 39 optimal well locations that have been identified have an associated quality of 158.135 and cost of 33.2.

The final results for each one of these regions are shown in Table 10. The optimal well locations and multiple completions are shown in Table 11 and illustrated in Figure 21.

Regarding the computational requirements, the commercially available software package CPLEX 4.08 is utilized for the solution of MILP mathematical models. Most of the time needed for the solution is devoted to the generation of the large number of clique constraints involved in the model. Using the branching strategy based on pseudo-reduced costs and the primal simplex approach to solve the subproblems at each node, the solution of the subproblems that correspond to each subregion within 0.1% integrality gap requires the solution of MILP problems that involve the constraints and variables shown in Table 12, where the B&B nodes and CPU times needed are also reported.

Table 6. Comparison with Proposed Well Locations Based on 70 Optimal Wells

Model	Quality
1	430,802.669
	92,097.307 (avg.)
2 (Euclidean)	98,089.269 (avg.)
2 (Rectilinear)	85,510.993 (avg.)
Proposed	170,647.038
	47,017.113 (avg.)

Table 7. Comparison with Proposed Well Locations Based on 53 Optimal Wells

Model	Avg. Quality	Quality Increase (%)
1	80,138.28	**70.445**
2 (Euclidean)	84,973.535	**80.729**
2 (Rectilinear)	77,384.721	**64.588**
Proposed	47,017.113	

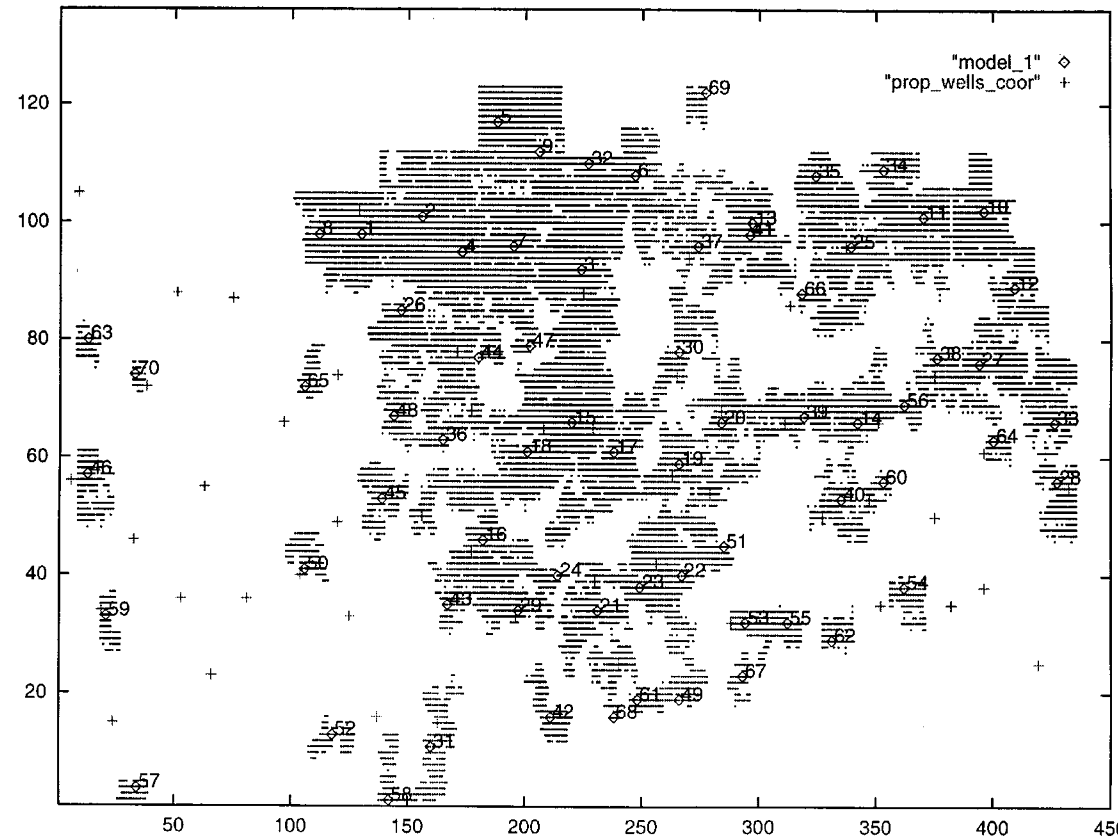

Figure 16. Model 1: comparison with proposed wells.

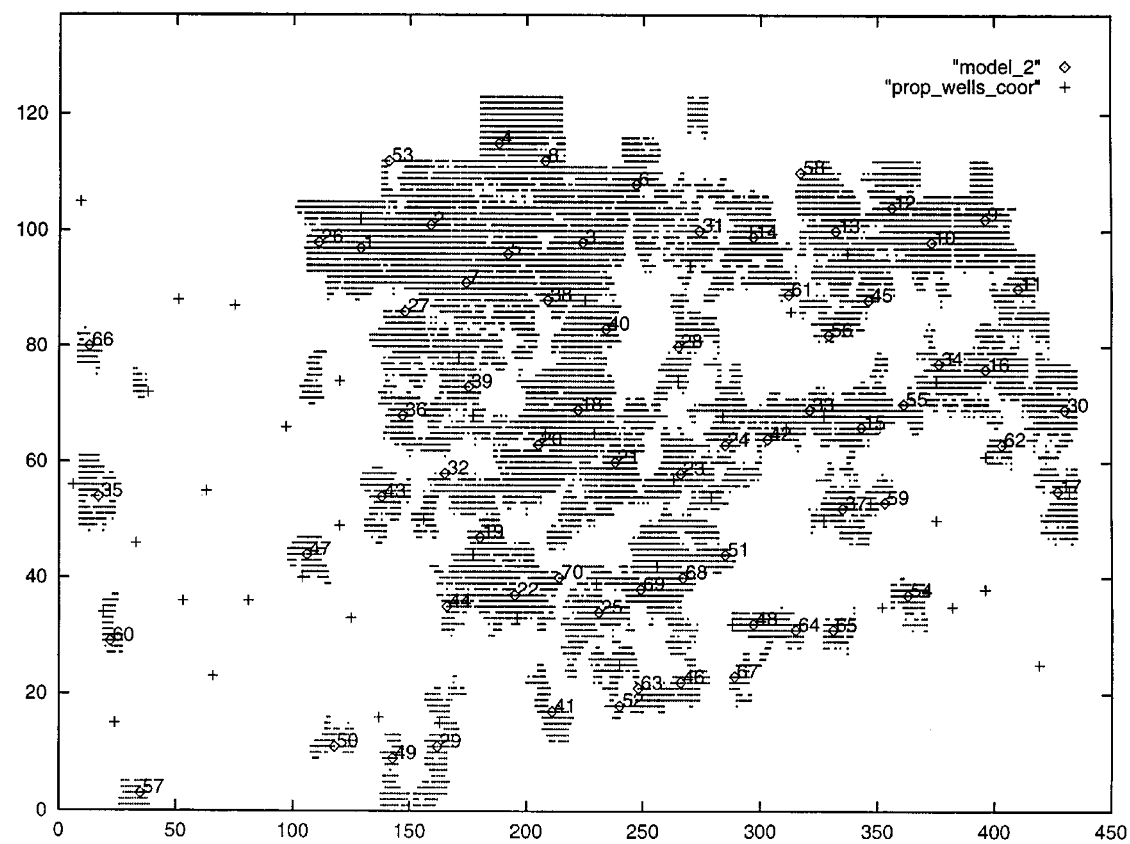

Figure 17. Model 2 with Euclidean distance constraints: comparison with proposed wells.

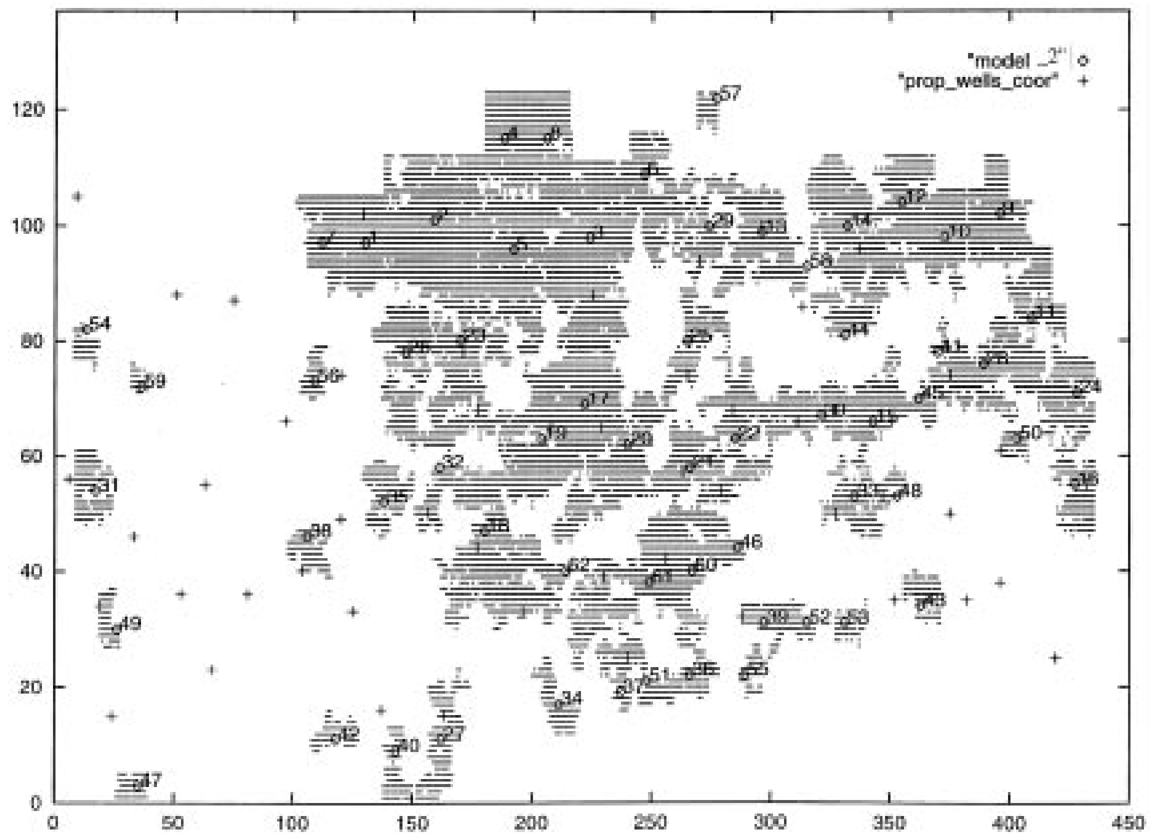

Figure 18. Model 2 with rectilinear distance constraints: comparison with proposed wells.

Decomposition Approach: Aggregated Distance Constraints. The alternative aggregated form of distance constraints (Eq. 1) are used for the solution of the optimization problems that correspond to the different subregions. The optimal well locations obtained are the same. However, the model size is drastically decreased, which results in a substantial improvement in the computational requirements of the solution pro-

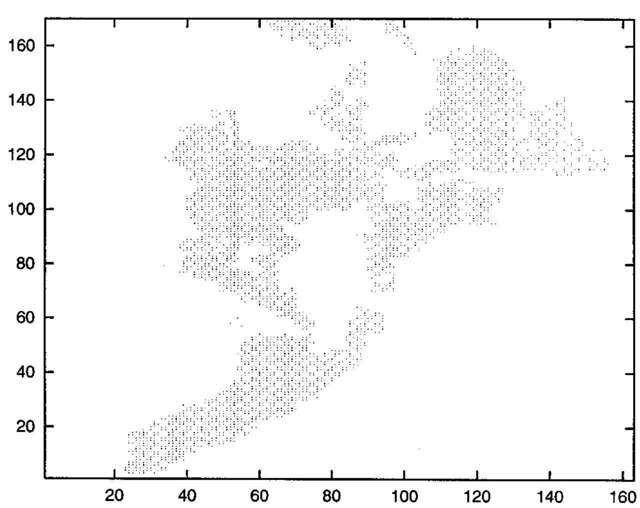

Figure 19. Potential well locations: Case Study 2.

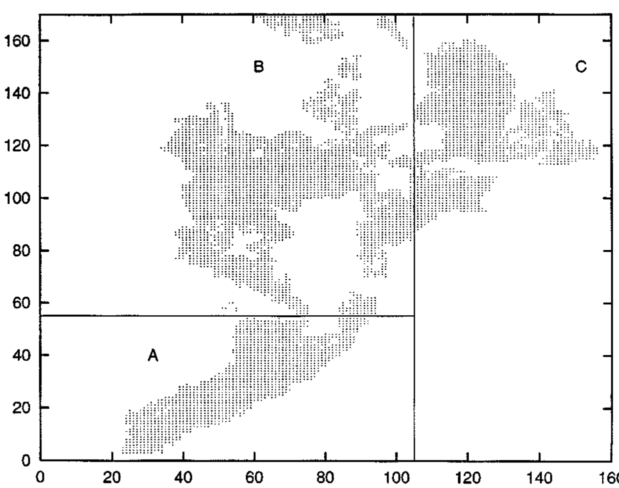

Figure 20. Field decomposition: Case Study 2.

Table 8. Boundaries of the Different Regions: Case Study 2

Region	x Bounds (Grid Units)	y Bounds (Grid Units)	No. of Potential Well Locations
A	$0 \leq x < 105$	$0 \leq y \leq 55$	1,359
B	$0 < x \leq 105$	$55 < y \leq 170$	3,167
C	$105 < x \leq 163$	$80 \leq y \leq 170$	1,680

Table 9. Results for Each Region

Region	Quality	Cost	Objective	No. of Wells
A	52.287	9.6	42.687	12
B	53.954	16.8	37.154	19
C	58.051	8.8	49.251	10
A + B + C	164.292	35.2	129.092	41

Table 10. Final Results for Different Regions

Region	Quality	Cost	Objective	No. of Wells
A	52.3	9.6	42.7	12
B	47.797	14.8	32.997	17
C	58.051	8.8	49.251	10
A + B + C	158.135	33.2	124.935	39

Table 11. Results of the Decomposition Approach

Well	X Coord.	Y Coord.	Quality	Geo-Object
1	131.0	133.0	12.466	1.0
2	113.0	121.0	11.184	1.0
3	113.0	139.0	9.374	1.0
4	57.0	35.0	7.742	3.0
5	131.0	115.0	6.979	1.0
6	54.0	24.0	5.108	5.0
7	72.0	40.0	4.479	5.0
8	66.0	109.0	2.590	4.0
9	56.0	17.0	5.851	3.0
10	75.0	33.0	5.468	3.0
11	66.0	119.0	5.434	2.0
12	54.0	42.0	5.026	5.0
13	131.0	151.0	4.779	1.0
14	66.0	101.0	4.770	2.0
15	84.0	118.0	4.387	2.0
16	84.0	104.0	0.908	4.0
17	48.0	118.0	3.900	2.0
	48.0	118.0	2.054	4.0
18	38.0	25.0	3.770	3.0
19	36.0	24.0	3.559	5.0
20	107.0	93.0	3.452	2.0
	107.0	93.0	0.881	9.0
21	37.0	7.0	3.412	3.0
22	47.0	100.0	3.159	2.0
	47.0	100.0	3.313	4.0
23	65.0	91.0	3.242	4.0
24	84.0	100.0	3.067	2.0
25	32.0	6.0	2.905	5.0
26	57.0	53.0	2.747	3.0
27	107.0	111.0	2.490	2.0
28	125.0	95.0	2.420	2.0
29	75.0	51.0	2.220	3.0
30	45.0	82.0	2.100	4.0
	45.0	82.0	0.917	6.0
31	113.0	157.0	1.946	1.0
32	53.0	136.0	1.673	4.0
33	62.0	64.0	1.164	6.0
34	144.0	130.0	1.131	8.0
	144.0	130.0	0.949	11.0
35	67.0	169.0	1.107	10.0
36	98.0	127.0	1.050	7.0
37	80.0	129.0	1.038	7.0
38	85.0	163.0	0.967	10.0
39	63.0	82.0	0.957	6.0

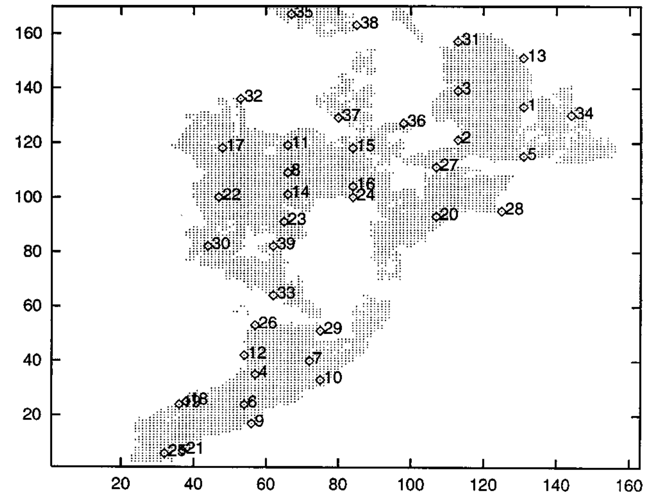

Figure 21. Model 3: optimal well locations—rectilinear distance.

cedure. The results obtained using CPLEX 6.0 in a HP C160 workstation are shown in Table 13.

Simultaneous Approach. In this section all potential well locations are considered simultaneously in an optimization problem of finding the optimal well locations and completions using the aggregated distance constraints. Forty wells are proposed with an associated quality of 158.85 and a cost of 34.0. The overall objective is 124.95. The results are shown in Table 14 and illustrated in Figure 22. The solution time for the simultaneous consideration of all the potential well locations was 12,532 CPU s using CPLEX 6.0 in a HP C160 workstation.

Comparison

As shown in Table 13, the decomposition approach requires the utilization of 1,402 CPU s for region A, 1,740 CPU s for region B, and 460 CPU s for region C using CPLEX 6.0 in a HP C160 workstation. In comparison to the solution of

Table 12. Computational Requirements for Different Regions

Region	Constraints	Binary Variables	Nodes	CPU (h)
A	322,439	3,804	31	37.8
B	423,477	5,495	4	30.3
C	211,785	2,834	2	5.6

Table 13. Computational Requirements for Different Regions Utilizing the Aggregated Distance Constraints

Region	Constraints	Binary Variables	Nodes	CPU (s)
A	8,921	3,494	3	1,402
B	17,104	7,020	2	1,741
C	8,319	3,394	2	460

Table 14. Results of the Simultaneous Consideration of All Potential Points

Well	ID/Geo-Obj	X Coord.	Y Coord.	Quality
1	W-131-115.G-0001	131.0	115.0	6.98
2	W-113-121.G-0001	113.0	121.0	11.18
3	W-131-133.G-0001	131.0	133.0	12.47
4	W-113-139.G-0001	113.0	139.0	9.37
5	W-131-151.G-0001	131.0	151.0	4.78
6	W-113-157.G-0001	113.0	157.0	1.95
7	W-107-093.G-0002	107.0	93.0	3.45
8	W-125-095.G-0002	125.0	95.0	2.42
9	W-047-100.G-0002	47.0	100.0	3.16
10	W-084-100.G-0002	84.0	100.0	3.07
11	W-066-101.G-0002	66.0	101.0	4.77
12	W-107-111.G-0002	107.0	111.0	2.49
13	W-048-118.G-0002	48.0	118.0	3.90
14	W-084-118.G-0002	84.0	118.0	4.39
15	W-066-119.G-0002	66.0	119.0	5.43
16	W-037-007.G-0003	37.0	7.0	3.41
17	W-056-017.G-0003	56.0	17.0	5.85
18	W-038-025.G-0003	38.0	25.0	3.77
19	W-075-033.G-0003	75.0	33.0	5.47
20	W-057-035.G-0003	57.0	35.0	7.74
21	W-075-051.G-0003	75.0	51.0	2.22
22	W-057-053.G-0003	57.0	53.0	2.75
23	W-045-082.G-0004	45.0	82.0	2.10
24	W-065-091.G-0004	65.0	91.0	3.24
	W-047-100.G-0004	47.0	100.0	3.31
25	W-084-104.G-0004	84.0	104.0	0.91
26	W-066-109.G-0004	66.0	109.0	2.59
	W-048-118.G-0004	48.0	118.0	2.05
27	W-053-136.G-0004	53.0	136.0	1.67
28	W-032-006.G-0005	32.0	6.0	2.91
29	W-036-024.G-0005	36.0	24.0	3.56
30	W-054-024.G-0005	54.0	24.0	5.11
31	W-072-040.G-0005	72.0	40.0	4.48
32	W-054-042.G-0005	54.0	42.0	5.03
33	W-062-064.G-0006	62.0	64.0	1.16
	W-045-082.G-0006	45.0	82.0	0.92
34	W-063-082.G-0006	63.0	82.0	0.96
35	W-098-127.G-0007	98.0	127.0	1.05
36	W-080-129.G-0007	80.0	129.0	1.04
37	W-144-130.G-0008	144.0	130.0	1.13
38	W-089-086.G-0009	89.0	86.0	0.82
	W-107-093.G-0009	107.0	93.0	0.88
39	W-085-162.G-0010	85.0	162.0	0.97
40	W-067-166.G-0010	67.0	166.0	1.11
	W-144-130.G-0011	144.0	130.0	0.95

the overall problem, the decomposition approach requires 3,603 CPU s using CLPEX 6.0 in a HP C160 workstation, compared to 12,532 CPU s required by the simultaneous approach. In terms of the quality of the solutions obtained, the decomposition approach results in the selection of 39 well sites with cost of 33.2 and overall quality of 158.135, which leads to an objective of 124.935. The simultaneous approach, on the other hand, places 40 wells with a cost of 34.0 and quality of 158.95, which in turn results in an objective of 124.95 very close to the objective value obtained from the decomposition approach. This is although the computational time required following the decomposition approach was one-fourth of the time needed for the simultaneous consideration of all the potential well locations.

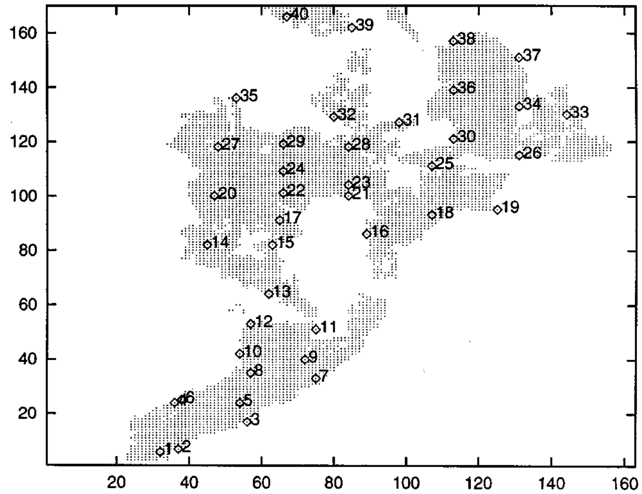

Figure 22. Model 3: optimal well locations—simultaneous approach.

Conclusions

This article proposed a novel decomposition-based approach to address the problem of field development for a given reservoir. The approach is based on:

- 3-D reservoir characterization by its quality and geo-object expressing field productivity and connectivity;
- field decomposition;
- an iterative scheme based on a quality cut-off criterion; and
- an efficient problem formulation that results in a MILP optimization problem that can be solved to optimality for each of the decomposed subproblems.

The proposed approach is applied to two industrial case studies involving 59,432 and 6,206 candidate surface well locations, respectively, for which there are 65 and 75 possible well completions.

Three alternative models are proposed to further exploit the nature of the problem. An important feature of Model 1 is that it can explicitly handle the grid points in the *z* direction and, consequently, can form the basis of placing deviated wells. The comparison of the determined well locations with the locations proposed following a heuristic procedure illustrates the advantage of the proposed approach.

Acknowledgments

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